MODEL OF THE MOTION AND SHOCK WAVES IN TWO-PHASE SOLIDS WITH PHASE TRANSITIONS

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The one-velocity and one-temperature model of the motion of a two-phase solid, in which each phase occupies a certain part of the volume, is considered. The investigation is carried out in Lagrangian variables, which offers certain advantages in solving one-dimensional nonstationary problems. The stress tensor for the mixture is decomposed into two parts — hydrostatic pressure, common to the two phases, associated with the three-term equation of state, and the deviator, which varies elastically up to a certain value and then remains constant. A certain relation, determined by the characteristic reaction time, is proposed for the phase transition kinetics. Then a solution is obtained for the problem of the nonstationary one-dimensional motion of a metal (iron) resulting from the impact of a plate against a target. The phase transitions ($Fe^{\alpha} \rightleftharpoons Fe^{\varepsilon}$) behind the wave and their characteristic time have an important effect on the damping of the disturbance and on the zone in which these transitions go to completion. A method is proposed for determining the coefficient in the relation for the phase transition rate from the residual effect (hardening) after impact.

1. We will examine in Lagrangian coordinates r^k the motion of a two-phase continuum in which each phase is characterized by its own true density $\rho_i{}^{\circ}$ and volume content α_i ($i = 1, 2$),

$$\alpha_1 + \alpha_2 = 1, \tag{1.1}$$

Let the velocities and hence the displacements of both phases coincide. We isolate a material volume V bounded by the material surface Σ. Considering its state at times $\tau = 0$ and $\tau = t$ in the absence of external body forces, we can write the integral mass-conservation equations for the first and second phases

$$\begin{aligned}\int_V \rho_{10}(r)\,dV &= \int_V \Big[\rho_1(r, t) I(r, t) + \int_0^t J_{12}(r, \tau) I(r, \tau)\,d\tau\Big] dV\\ \int_V \rho_{20}(r)\,dV &= \int_V \Big[\rho_2(r, t) I(r, t) - \int_0^t J_{12}(r, \tau) I(r, \tau)\,d\tau\Big] dV.\end{aligned} \tag{1.2}$$

The subscripts 1 and 2 relate to the first and second phases, respectively; the subscript 0 relates to the initial state at $\tau = 0$; J_{12} is the phase transition rate or the mass transferred from the first phase to the second (or conversely, with opposite sign) per unit volume of mixture per unit time; I(r,t) is the degree of expansion of the medium or the Jacobian of the transformation from Eulerian to Lagrangian coordinates; ρ_i is the mean density of the i-th phase;

$$\rho_1 = \rho_1^{\circ}\alpha_1, \quad \rho_2 = \rho_2^{\circ}\alpha_2. \tag{1.3}$$

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Adding Eqs. (1.2), we obtain the mass-conservation equation for the mixture:

$$\int_V \rho_0(r)\,dV = \int_V \rho(r,\ t)\,I(r,\ t)\,dV \qquad (\rho = \rho_1 + \rho_2)\,. \tag{1.4}$$

(ρ – density of mixture)

Within the framework of the one-velocity and one-temperature model it is sufficient to consider only the equations of motion and energy of the mixture as a whole:

$$\begin{gathered}\int_V \rho_0(r)\,[\mathbf{v}(r,\ t) - \mathbf{v}_0(r)]\,dV = \int_0^t\!\!\int_\Sigma \sigma_n{}^\circ d\sum d\tau \\ \int_V \left[(\rho_1 e_1 + \rho_2 e_2)\,I - (\rho_{10}e_{10} + \rho_{20}e_{20}) + \rho_0\frac{v^2 - v_0{}^2}{2}\right] dV = \int_0^t\!\!\int_\Sigma \sigma_n{}^\circ \mathbf{v}\,d\sum d\tau\,.\end{gathered} \tag{1.5}$$

Here $\mathbf{v}$ is the velocity of the medium, e_i the specific internal energy of the i-th phase, and σ° the Lagrange stress tensor [1] in the medium in question.

Using the Gauss theorem and differentiating (1.2) and (1.5) with respect to t, after certain transformations we obtain the following system of differential equations:

$$\begin{gathered}\frac{\rho_0}{\rho}\frac{\partial\rho_1}{\partial t} + \rho_1\frac{\partial I}{\partial t} + \frac{\rho_0}{\rho}J_{12} = 0, \quad \frac{\rho_0}{\rho}\frac{\partial\rho_2}{\partial t} + \rho_2\frac{\partial I}{\partial t} - \frac{\rho_0}{\rho}J_{12} = 0 \\ \rho_0\frac{\partial v}{\partial t} = \nabla\cdot\sigma^\circ, \quad \frac{\rho_0}{\rho}\left[\rho_1\frac{\partial e_1}{\partial t} + \rho_2\frac{\partial e_2}{\partial t} + (e_2 - e_1)\,J_{12}\right] = (\sigma^\circ\cdot\nabla)\,\mathbf{v}\,.\end{gathered} \tag{1.6}$$

2. We must now consider the thermodynamic and mechanical properties of the medium. The Euler stress tensor σ^{kl} can be represented in the form of a sum of isotropic and deviatoric components,

$$\sigma^{kl} = -p\delta^{kl} + \tau^{kl}\,. \tag{2.1}$$

We take the value of p from the equations of state, assuming that both phases are at the same pressure:

$$p = p_1(\rho_1{}^\circ,\ T) = p_2(\rho_2{}^\circ,\ T)\,. \tag{2.2}$$

The stress deviator for the mixture is

$$\tau^{kl} = \alpha_1\tau_1{}^{kl} + \alpha_2\tau_2{}^{kl}\,. \tag{2.3}$$

We assume that the stress deviator of the i-th phase $\tau_i{}^{kl}$ varies elastically up to the yield point $\tau^*{}_i$, after which it should remain constant. We can then write the following equation for the deviator components in the principal axes:

$$\frac{\partial\tau_i^{kl}}{\partial t} = 2\mu_i\left(\varepsilon_i{}^{kl} + \frac{1}{3\rho}\frac{\partial\rho}{\partial t}\right)\delta^{kl} \qquad (\tau_i{}^{kl} \leqslant \tau_i{}^*) \tag{2.4}$$

where $\varepsilon_i{}^{kl}$ is the strain rate tensor. We note that in the presence of impact loading pressures of the order of hundreds of kilobars the deviator, despite its smallness as compared with the hydrostatic pressure, may have an influence on the damping of the disturbance.

We will not consider questions relating to the use of the Lagrange stress tensor, since in what follows the equations obtained are used to solve one-dimensional nonstationary problems, in which these questions do not present difficulties.

The specific internal energy and pressure of a solid or liquid are usually represented in the form of a sum of three components [2-4], which respectively describe the cold elastic properties, the harmonic vibrations of the lattice atoms, and the thermal excitation of the electrons together with anharmonic atomic vibration effects

$$e(\rho^\circ,\ T) = e_p + e_T + e_{ea}, \qquad p(\rho^\circ,\ T) = p_p + p_T + p_{ea}$$

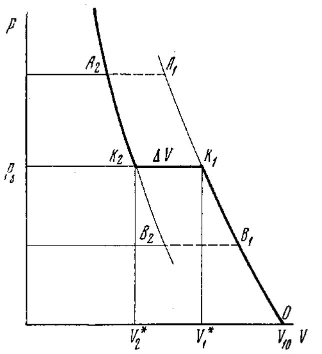

Fig. 1

$$e_p = \int_{\rho_{00}^\circ}^{\rho^\circ} \frac{p_p(\rho^\circ)}{\rho^{\circ 2}} d\rho^\circ \quad \left(p_p(\rho^\circ) = \rho^{\circ 2} \frac{de_p}{d\rho^\circ}\right)$$
$$e_T = cT, \qquad p_T = \gamma(\rho^\circ)\rho^\circ e_T, \qquad e_{ea} = B(\rho^\circ)T^2, \qquad p_{ea} = \Gamma(\rho^\circ)\rho^\circ e_{ea}\,. \tag{2.5}$$

Here, $\gamma(\rho^\circ)$ is the Grüneisen constant (for harmonic atomic vibrations), c = const is the atomic heat, $\Gamma(\rho^\circ)$ is an analogue of the Grüneisen constant associated with anharmonic effects and electron excitation. The function $p_p(\rho^\circ)$, on which $B(\rho^\circ)$, $\Gamma(\rho^\circ)$ and $\gamma(\rho^\circ)$ depend, is determined experimentally. A very convenient method of determining the equation of state of metals and ionic crystals, the so-called method of potentials, was proposed in [4]. The potential components for the internal energy and pressure have the form

$$e_p(\rho^\circ) = 3Ab^{-1}\exp b(1 - x^{1/3}) - 3Kx^{-1/3} \qquad (x = \rho_0^\circ / \rho^\circ)$$
$$p_p(\rho^\circ) = Ax^{-2/3}\exp b(1 - x^{1/3}) - Kx^{-4/3}\,. \tag{2.6}$$

Here, A, K, b, ρ_0° are quantities fixed for each metal or each phase. For the Grüneisen constant it is possible to take the linear approximation

$$\gamma(\rho^\circ) = \alpha - \beta\rho^\circ / \rho_0^\circ \tag{2.7}$$

which is valid for many metals and their phases over a quite broad density range.

The approximation of constant specific heat for e_T significantly reduces the volume of computations and at the same time is accurate enough at $T > \theta$, where θ is the Debye temperature, and is perfectly justifiable in connection with impact loading problems.

The part of the internal energy associated with the square of the temperature becomes important when the pressure in the shock wave exceeds 1 mbar (T > 10,000°K). There are varying opinions [2-4] regarding the computation of e_{ea} and p_{ea}.

The possibility of phase transitions leads to the necessity of coordinating the equations of state of the phases for the internal energies, in order to take the energy transitions correctly into account. We introduce the operator

$$E_i(\rho_i^\circ, T) = e_{pi}(\rho_i^\circ) + c_iT + B_i(\rho_i^\circ)T^2. \tag{2.8}$$

Then the normalized equations for the internal energies of the phases can be written in the form

$$e_1(\rho_i^\circ, T) = E_1(\rho_1^\circ, T) + e_{01}, \qquad e_2(\rho_2^\circ, T) = E_2(\rho_2^\circ, T) + e_{02}$$
$$(e_{01} = \text{const},\ e_{02} = E_1(\rho_{1s0}^\circ, T_0) + l_e(T_0) - E_2(\rho_{2s0}^\circ, T_0) + e_{01} = \text{const}) \tag{2.9}$$

where the phase densities on the saturation line ρ_{1S}° and ρ_{2S}° are determined from the equations

$$p_1(\rho_{1s0}^\circ, T_0) = p_s(T_0), \qquad p_2(\rho_{2s0}^\circ, T_0) = p_s(T_0) \tag{2.10}$$

where $p_S(T)$ is the phase transition pressure or the phase equilibrium line, and the p_i take the form

$$p_i(\rho_i, T) = p_{pi}(\rho_i^\circ) + \gamma_i(\rho_i^\circ)\rho_i^\circ c_iT + B_i(\rho_i^\circ)\Gamma(\rho_i^\circ)\rho_i^\circ T^2. \tag{2.11}$$

If the equations of state of the phases (2.11) and the saturation line $p_S(T)$ are known, starting from the Clapeyron-Clausius equation we can determine

$$l_e(T) = \left(\frac{1}{\rho_{2s}^\circ} - \frac{1}{\rho_{1s}^\circ}\right)\left(T\frac{dp_s}{dT} - p_s\right). \tag{2.12}$$

Generally speaking, the function $p_S(T)$ and the equations of state of the phases (2.9) and (2.11) are not independent and they must be adjusted so that the Clapeyron-Clausius condition is satisfied on the entire saturation curve.

The phase transition kinetics in solids at high pressures have not been investigated in sufficient detail from the quantitative standpoint. Therefore, as a first approximation, it is desirable to start from the general propositions of the thermodynamics of irreversible processes. First, we divide J_{12} into two components

$$J_{12} = j_{12} - j_{21} \tag{2.13}$$

each of which can only be nonnegative ($j_{12} \geq 0$, $j_{21} \geq 0$), while at any point one of them is necessarily equal to zero. If there are no phase transitions, then $j_{12} = j_{21} = 0$. Then j_{12} gives the rate of transition from the first phase to the second, and j_{21} that from the second to the first. We assume that the phase transition (for example, 1 → 2, if $\rho_1 > 0$) proceeds the more rapidly, the more the thermodynamic potential of the substance in the first phase exceeds that of the substance in the second phase at the same pressure and temperature (and similarly for the transition 2 → 1, if $\rho_2 > 0$), i.e.,

$$\begin{gathered}
j_{12} = F_{12}\,[\varphi_1(p,\ T) - \varphi_2(p,\ T)] \quad (\rho_1 > 0,\ \varphi_1 > \varphi_2), \\
j_{12} = 0 \quad (\rho_1 = 0 \ \text{ or } \varphi_1 \leqslant \varphi_2) \\
j_{21} = F_{21}\,[\varphi_2(p,\ T) - \varphi_1(p,\ T)] \quad (\rho_2 > 0,\ \varphi_2 > \varphi_1), \\
j_{21} = 0 \quad (\rho_2 = 0 \ \text{ or } \varphi_2 \leqslant \varphi_1)\ .
\end{gathered} \tag{2.14}$$

We note that

$$\varphi_1(p_s(T),\ T) = \varphi_2(p_s(T),\ T)\ . \tag{2.15}$$

Let us examine the pV diagram (Fig. 1), where we have plotted the isotherms corresponding to the first phase OK_1A_1 and the second phase $B_2K_2A_2$ for the same temperature.

Along the isotherm we have $d\varphi = Vdp$ ($\varphi = e + pV - Ts$, where s is the entropy and V the specific volume). Hence it is clear that the differences of the thermodynamic potentials at points A_1 and A_2 and at points B_1 and B_2 are equal to the areas $K_1K_2A_2A_1$ and $K_1K_2B_2B_1$, respectively, or

$$\varphi(A_1) - \varphi(A_2) \approx (p - p_s)\Delta V, \quad \varphi(B_2) - \varphi(B_1) \approx (p_s - p)\Delta V.$$

Then equations (2.13) take the form

$$\begin{gathered}
j_{12} = F_{12}\Delta V\,(p - p_s) \quad (\rho_1 > 0,\ p > p_s) \\
j_{12} = 0 \quad (\rho_1 = 0 \ \text{ or } \ p \leqslant p_s) \\
j_{21} = F_{21}\Delta V\,(p_s - p) \quad (\rho_2 > 0,\ p_s > p) \\
j_{21} = 0 \quad (\rho_2 = 0 \ \text{ or } \ p_s \leqslant p).
\end{gathered} \tag{2.16}$$

Starting from the characteristic transition time, we can estimate the coefficients F_{12} and F_{21}. We note that a similar investigation was conducted in [5] for the phase transition kinetics relating to vapor-liquid mixtures.

3. We write the system of differential equations (1.6) for one-dimensional plane motion ($v_1 = v(r)$, $v_2 = v_3 = 0$), going over to the variables $\rho_1°$, $\rho_2°$, α, v, T, p, τ^{11}, r, where r is the Lagrangian coordinate in the direction of motion. In this connection we make use of the fact that

$$\partial I / \partial t = \partial v / \partial r \tag{3.1}$$

and the fact that the components of the Lagrangian $\sigma°$ and Eulerian σ stress tensors are related by the following expressions:

$$\sigma^{\circ 11} = \sigma^{11}, \qquad \rho\,\sigma^{\circ 22} = \rho\,\sigma^{\circ 33} = \rho_0\sigma^{22} = \rho_0\sigma^{33}\ . \tag{3.2}$$

Then system (1.6) takes the form

$$\begin{gathered}
\frac{\alpha}{\rho}\frac{\partial \rho_1°}{\partial t} + \frac{\rho_1°}{\rho}\frac{\partial \alpha}{\partial t} = b_1 = -\frac{\rho_1}{\rho_0}\frac{\partial v}{\partial r} - \frac{J_{12}}{\rho} \\
\frac{1-\alpha}{\rho}\frac{\partial \rho_2°}{\partial t} - \frac{\rho_2°}{\rho}\frac{\partial \alpha}{\partial t} = b_2 = -\frac{\rho_2}{\rho_0}\frac{\partial v}{\partial r} + \frac{J_{12}}{\rho}
\end{gathered}$$

$$
\begin{aligned}
&a_{31}\frac{\partial \rho_1{}^\circ}{\partial t}+a_{32}\frac{\partial \rho_2{}^\circ}{\partial t}+a_{33}\frac{\partial T}{\partial t}=b_3=\frac{\sigma^{11}}{\rho_0}\frac{\partial v}{\partial r}-(e_2-e_1)\frac{J_{12}}{\rho}\\
&a_{41}\frac{\partial \rho_1{}^\circ}{\partial t}+a_{42}\frac{\partial \rho_2{}^\circ}{\partial t}+a_{43}\frac{\partial T}{\partial t}=0, \qquad \frac{\partial v}{\partial t}=\frac{1}{\rho_0}\frac{\partial \sigma^{11}}{\partial r}\ .
\end{aligned}
\tag{3.3}
$$

Here,

$$
\begin{aligned}
&a_{31}(\rho_1{}^\circ,\ \rho_2{}^\circ,\ \alpha,\ T)=\frac{\rho_1}{\rho}\left(\frac{\partial E_1}{\partial \rho_1{}^\circ}\right)_T, \qquad a_{32}(\rho_1{}^\circ,\ \rho_2{}^\circ,\ \alpha,\ T)=\frac{\rho_2}{\rho}\left(\frac{\partial E_2}{\partial \rho_2{}^\circ}\right)_T\\
&a_{33}(\rho_1{}^\circ,\ \rho_2{}^\circ,\ \alpha,\ T)=\frac{\rho_1}{\rho}\left(\frac{\partial E_1}{\partial T}\right)_{\rho_1{}^\circ}+\frac{\rho_2}{\rho}\left(\frac{\partial E_2}{\partial T}\right)_{\rho_2{}^\circ}, \qquad a_{41}(\rho_1{}^\circ,\ T)=\left(\frac{\partial p_1}{\partial \rho_1{}^\circ}\right)_T\\
&a_{42}(\rho_2{}^\circ,\ T)=-\left(\frac{\partial p_2}{\partial \rho_2{}^\circ}\right)_T, \qquad a_{44}(\rho_1{}^\circ,\ \rho_2{}^\circ,\ T)=\left(\frac{\partial p_1}{\partial T}\right)_{\rho_1{}^\circ}-\left(\frac{\partial p_2}{\partial T}\right)_{\rho_2{}^\circ}.
\end{aligned}
\tag{3.4}
$$

The fourth of Eqs. (3.3) was obtained after differentiating Eq. (2.2) with respect to t (equal phase pressures). Adding the first two of Eqs. (3.3), we obtain the mixture continuity equation

$$
\frac{1}{\rho}\frac{\partial \rho}{\partial t}=-\frac{\rho}{\rho_0}\frac{\partial v}{\partial r}\ .
\tag{3.5}
$$

For the case in question, Eq. (2.4) for the deviator component $\tau_i{}^{11}$, with account for (3.5), takes the form

$$
\frac{\partial \tau_i{}^{11}}{\partial t}=\frac{4}{3}\mu\frac{\rho}{\rho_0}\frac{\partial v}{\partial r} \qquad (\tau_i{}^{11}\leqslant \tau_i{}^*)\ .
\tag{3.6}
$$

For given phase equations of state (2.9), (2.11), phase equilibrium line $p_s(T)$, and coefficients F_{12} and F_{21} in Eqs. (2.16) system (3.3), together with (3.4), (2.1), (3.6), is closed in the region of continuous motion of a two-phase compressible solid. We note that in the case of absence of one of the phases ($\alpha \equiv 0$ or $\alpha \equiv 1$) system (3.3) automatically goes over into the system of equations of motion of the corresponding one-phase medium.

We have examined, within the framework of the model described, the problem of the plane impact of a plate of thickness l_1 (projectile), with infinite dimensions in a direction perpendicular to the motion, on a half-space or layer of thickness $l-l_1$ (target, located on the right). Let the stresses (boundary conditions) to the left of the point $r=0$ and to the right of the point $r=l$ be known

$$
\sigma^{11}(0,\ t)=\sigma_0(t), \qquad \sigma^{11}(l,\ t)=\sigma_l(t),
\tag{3.7}
$$

In the simplest case $\sigma_0(t)\equiv\sigma_l(t)\equiv 0$. At $t=0$ the undisturbed state of the system (initial conditions) is known

$$
\rho_{10}{}^\circ,\ \rho_{20}{}^\circ, \qquad \alpha_0\equiv 1, \qquad T_0, \qquad \tau_0{}^{11}\equiv 0
$$

and, moreover,

$$
v_0(r)\equiv v_0 \quad (0\leqslant r\leqslant l_1), \qquad v_0(r)\equiv 0 \quad (l_1\leqslant r\leqslant l)\ .
\tag{3.8}
$$

In order to solve the problem we employed the so-called method of straight lines or the particle method with pseudo-viscosity.

In [6] this method was examined, with appropriate references, in connection with the calculation of elastoplastic flows in Euler variables.

In the coordinate system (r,t) we take points $r_0, r_1, \ldots, r_{n-1}, r_n$ ($r_0=-r_1$, $r_0<0$; $r_n=l+l-r_{n-1}$, $r_{n-1}<l$) on the r axis in the flow region and draw the straight lines $r=r_j$ ($j=0, 1, \ldots, n$), which corresponds to dividing the system into n material particles. Assuming the existence of a sufficiently smooth solution, we determine the values of the unknown functions at points $r=r_j$, and substitute for the derivatives with respect to r the difference relations

$$\left(\frac{\partial v}{\partial r}\right)_j = \frac{v_{j+1}(t) - v_{j-1}(t)}{r_{j+1} - r_{j-1}}, \quad \left(\frac{\partial \sigma^{11}}{\partial r}\right)_j = \frac{\sigma^{11}_{j+1}(t) - \sigma^{11}_{j-1}(t)}{r_{j+1} - r_{j-1}}, \tag{3.9}$$

the boundary conditions (3.7) being removed to $r = r_0$ and $r = r_n$, respectively. As a consequence, the system of six partial differential equations (3.3) and (3.6) is converted into a system of 6n ordinary differential equations with respect to time, which together with the initial conditions at $t = 0$ give a Cauchy problem whose computer solution does not present special difficulties.

When there are shock waves in the flow region, the solution of the problem should have a discontinuity, at which the Hugoniot conditions must be satisfied. In order to apply the method described to these problems, Richtmyer and Neumann proposed introducing the pseudo-viscosity, whereupon the stress tensor takes the form

$$\begin{gathered} \sigma_j^{11} = -p_j + \tau_j^{11} - \varphi_j \\ \varphi_j = \rho_0 a_0^{2-n} C_n \left| v_{j+1} - v_j \right|^n \quad (v_{j+1} - v_j < 0), \qquad \varphi_j = 0 \quad (v_{j+1} - v_j \geqslant 0) \end{gathered} \tag{3.10}$$

the linear ($n = 1$, $C_1 \approx 1$) and quadratic ($n = 2$, $C_2 \approx 2$) pseudo-viscosities being usually employed.

We note that the stress at a point separating the projectile and the target cannot be tensile; therefore, starting from time t_1 (after passage of the shock wave), when

$$\sigma^{11}(l_1, t_1) = -p(l_1, t_1) + \tau^{11}(l_1, t_1) = 0$$

we obtain the additional boundary condition

$$\sigma^{11}(l_1, t) = 0 \qquad (t \geqslant t_1) \tag{3.11}$$

which corresponds to the independent motion of the target and the projectile (withdrawal or rebound of the projectile). A similar situation may develop after spallation (disturbance of continuity in the interior of the specimen). However, the question of the onset of spallation [7] is a rather complex one and will not be considered here, although mathematically it is not difficult to take spallation effects into account.

4. The above model and calculation scheme were used to investigate nonstationary motion, when the material of the projectile and the target is iron, in which the transition $Fe^{\alpha} \rightarrow Fe^{\varepsilon}$ takes place behind a sufficiently intense shock, and during unloading the transition $Fe^{\varepsilon} \rightarrow Fe^{\alpha}$. In the undisturbed state only the first (Fe^{α}) phase is present ($\alpha_0 \equiv 1$, $\rho_0 = 7860$ kg/m^3). The coefficients of equations of state (2.5)-(2.7) have the following values [4]:

for Fe^{α}
$A = 9.9743 \cdot 10^5$ bar, $K = 10.1639 \cdot 10^5$ bar, $b = 7.0985$
$c = 4.45 \cdot 10^2$ m^2/sec^2·deg $\gamma_1 = 2.04 - 0{,}36\ \rho_1^{\circ} / \rho_0$,
for Fe^{ε}
$A = 9.4389 \cdot 10^5$ bar, $K = 10.740 \cdot 10^5$ bar, $b = 7.7845$
$c = 4.45 \cdot 10^2$ m^2/sec^2·deg, $\gamma_2 = 2.45 - 0{,}77\ \rho_2^{\circ} / \rho_0$

The phase equilibrium curve $p_s(T)$ for Fe^{α} and Fe^{ε} can be represented in the form

$$\begin{gathered} p_s(T) = K_0\,[0{,}0901 - 0{,}0152\,(T / T_0) + 0.0021\,(T / T_0)^2] \\ (K_0 = 16.953 \cdot 10^5\ бар,\ T_0 = 300^{\circ}\ \mathrm{K}). \end{gathered} \tag{4.1}$$

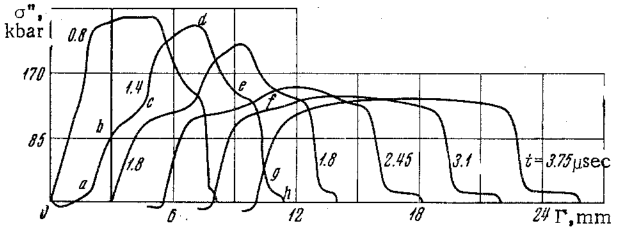

Fig. 2

This relation corresponds to the curve obtained by Kaufman [8] as a result of an analysis of the static and dynamic experiments of various authors and located on the phase diagram of of iron at temperatures below the so-called triple point (p = 110 kbar, T = 800° K). Certain aspects of this phase transition in iron were examined in [2-4] with references to the work of S. A. Novikov, A. G. Ivanov et al., A. Bolchan, and D. Erkman et al. in

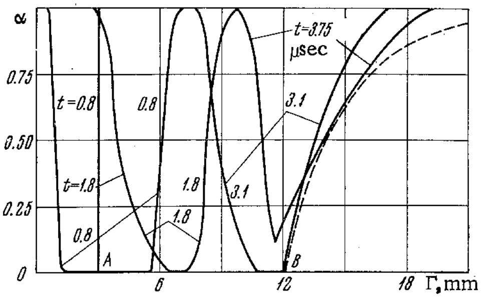

Fig. 3

which a series of characteristics of shock waves in solids that experience phase transitions (in particular, the formation of a multiwave structure and unloading waves) were examined, chiefly for the stationary case.

For lack of other data the quantities characterizing the resistance of the medium to dynamic shear were assumed to be the same for both phases

$$\mu_1 = \mu_2 = 0{,}906 \cdot 10^6 \text{ bar}, \; \tau_1^* = \tau_2^* = 4.79 \cdot 10^3 \text{ bar}, \;.$$

By way of example Figs. 2 and 3 show diagrams of the stresses σ^{11} and α at various moments of time for a plate ($l_1 = 3$ mm) striking a target with velocity $v_0 = 1325$ m/sec. The values $F_{12} = F_{21} = 6.45$ sec/m^2 were taken for the phase transition rate coefficients. The curve corresponding to t = 1.4 μsec (Fig. 2) is denoted by letters, hg being the elastic precursor, gf, the jump taking the first phase into the nonequilibrium state (of finite thickness owing to the pseudo-viscosity), ed, the relaxation zone or phase transition wave, where the transition $Fe^{\alpha} \rightarrow Fe^{\varepsilon}$ takes place (its thickness is determined by F_{12}), dcb, the unloading wave taking the second phase into the nonequilibrium state, ba, the unloading wave corresponding to the transition $Fe^{\varepsilon} \rightarrow Fe^{\alpha}$ (if $F_{21} \rightarrow \infty$, ba is transformed into an unloading shock); ao is the next unloading wave.

The presence of a deviator leads to the earlier onset of shock-wave attenuation as compared with the purely hydrodynamic model, since the elastic unloading wave has a greater velocity than the plastic unloading wave. The finite time required for phase transition and the multiwave structure formed also result in the wave, at which the transition $Fe^{\alpha} \rightarrow Fe^{\varepsilon}$ ends, beginning to attenuate much earlier than follows from simple considerations associated with the shock adiabat. Moreover, at the moment of impact the shock amplitude corresponds to the shock compression of the first phase, in which the entire particle velocity drop (in the given case $v_0/2$) is realized. With distance from the impact point the amplitude of this jump tends to its stationary value, when it corresponds to only part of the drop, the remainder corresponding to the phase transition wave.

We note that if the impact velocity v_0 of a plate of the same material and sufficient thickness exceeds 1625 m/sec, the structure of the entire shock wave tends to the stationary configuration before the arrival of the unloading wave.

If $650 < v_0 < 1625$ m/sec, then this stationary configuration does not exist, since the jump gf has a greater velocity than the wave ed, but each of these waves tends to its stationary amplitude before the arrival of the unloading wave.

5. Calculations with different values of the coefficient F_{12} for the rate of the phase transition 1 → 2 [see (2.16)] show that, other things being equal, the depth δ of the zone in which the phase transitions were completed (AB in Fig. 3), depends importantly on the assumed value of F_{12}, while the curve $\alpha_{\min}(r)$ (dashed line in Fig. 3) rises steeply from the point B. This makes it possible to determine F_{12} from the residual effect, for which it is necessary to determine the actual depth of the zone δ after a suitable experiment.

In this connection we draw attention to the "explosion hardening" of iron and low-carbon steel, when the passage of a shock wave of sufficient intensity is followed by a significant increase in the mechanical characteristics (for example, hardness) [9]. An analysis of the hardness curves over the depth of the target shows that at sufficiently high velocities there are three hardening zones: a first zone of substantial hardening, in which the hardness is actually constant over the depth; a second narrow zone of moderate hardening, in which the hardness falls sharply with depth, and a third zone of relatively slight hardening, in which the hardness falls gradually with depth. The depth of the first zone is determined by the impact velocity.

These facts suggest that hardening in the first and second zones is associated with the phase transition $Fe^{\alpha} \rightleftharpoons Fe^{\varepsilon}$, and hardening in the third zone with plastic deformation in the shock wave, whose pressure is now lower than that corresponding to phase transition. In the first zone the phase transitions are complete, in the second zone only partial. In this case in order to determine the depth of the zone AB it is obviously sufficient to determine in each instance the depth δ_e of the zone in which the hardness is constant after hardening as a function of the impact velocity at a fixed projectile thickness. If it is possible to select a value of F_{12} such that $\delta(v_0) \equiv \delta_e(v_0)$, then this confirms the hardening picture indicated and the proposed relation for the phase transition kinetics (2.16), as well as giving the real value of F_{12}.

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